DesEng Mathematics

Chapter 1 – Taming Trigonometry and Triangles

Trigonometry / Pythagoras: Checklist

- Know and apply the trigonometric ratios, $\sin\theta$, $\cos\theta$ and $\tan\theta$ and apply them to find angles and lengths in right-angled triangles in 2D figures.
- Apply the trigonometry of right-angled triangles in more complex figures, including 3D figures.
- Know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0°, 30°, 45°, 60°$ and $90°$. Know the exact value of $\tan\theta$ for $\theta = 0°, 30°, 45°$ and $60°$.
- Know and apply the **sine rule** to find lengths and angles.
- Know and apply the **cosine rule** to find lengths and angles.

Trigonometry / Triangles: Checklist

Pythagoras' Theorem: $a^2 + b^2 = c^2$

Sine, Cosine, Tangent: $\sin\theta$ = Opposite / Hypotenuse

$\cos\theta$ = Adjacent / Hypotenuse

$\tan\theta$ = Opposite / Adjacent

Sine rule: $a / \sin A = b / \sin B = c / \sin C$

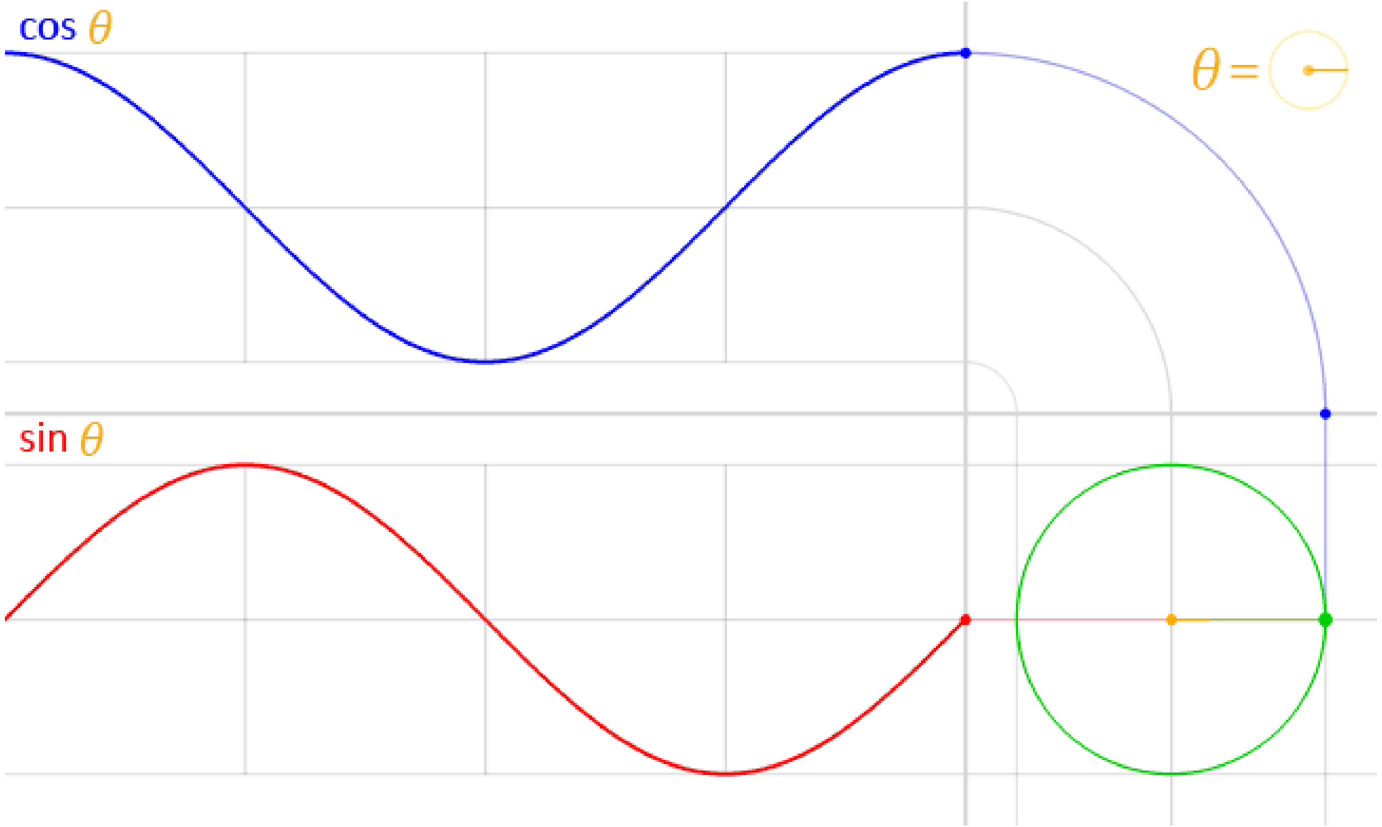
Cosine rule: $a^2 = b^2 + c^2 - 2bc.\cos A$

Know how to obtain exact values for sin/cos 0, 30, 45, 60, 90

Trigonometry: Standard Results 1

As the angle θ changes, the red point moves up and down, tracing the red graph. This is the graph for the sine function.

As the angle θ changes, the blue point moves left and right, tracing the blue graph. This is the graph for the cosine function.

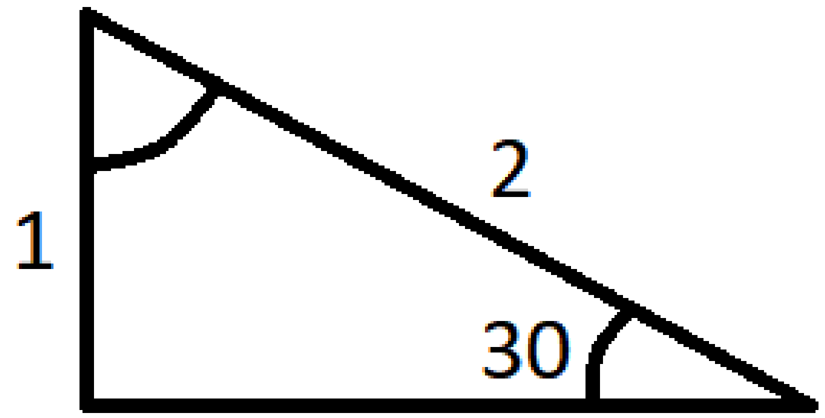


sin0° = 0	cos0° = 1
sin90° = 1	cos90° = 0
sin180° = 0	cos180° = -1
sin270° = -1	cos270° = 0
Sin360 = 0	cos360° = 1

Note: tanθ = sinθ / cos θ

Trigonometry: Standard Results 2

Start with a 30 ° right-angled triangle.



Trigonometry: Standard Results 2

Start with a 30 ° right-angled triangle.

You can then derive results for 30° and 60° using Pythagoras'.

$\sin 30° = ½$

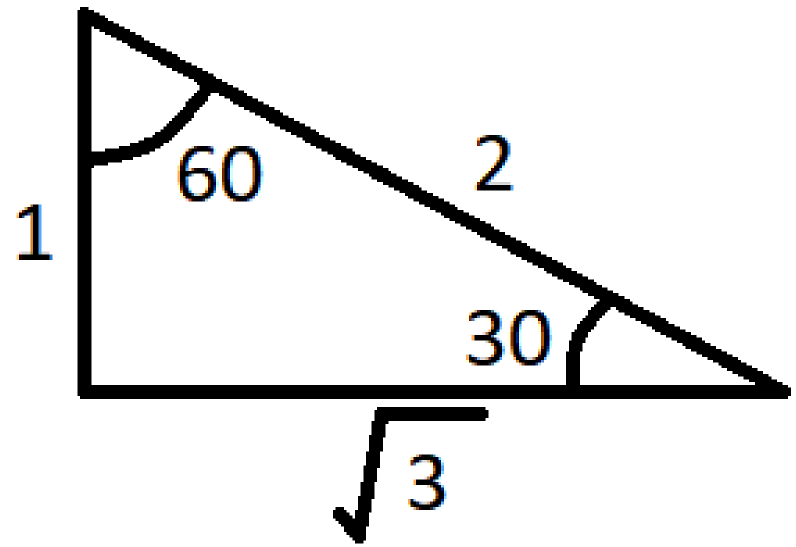
$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = 1/2$

$\tan 60° = \sqrt{3}$



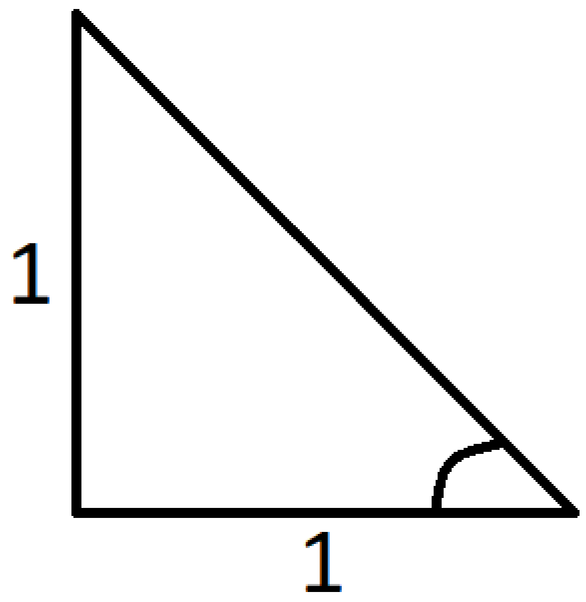
Pro tip: If you're doing A-Level maths, you'll have your calculator set to radians sometimes.

Always check that your calculator tells you that <u>sin30 is 0.5</u> before doing DesEng calculations.

If it doesn't, you'll need to set your calculator back to degrees.

Trigonometry: Standard Results 3

Start with a 45 degree right-angled triangle…



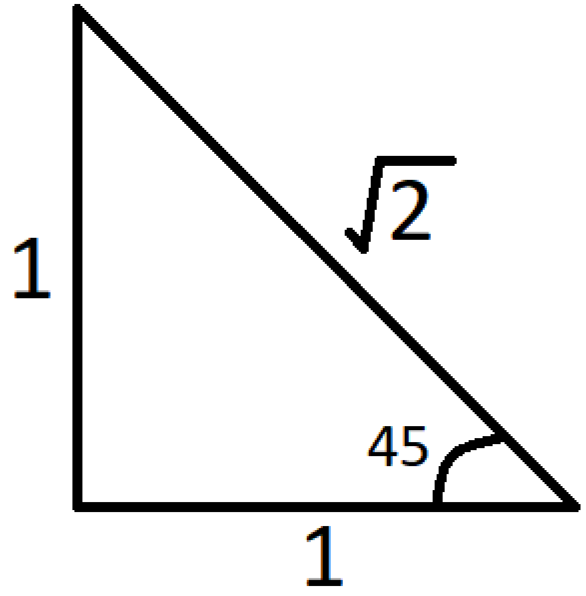
Trigonometry: Standard Results 3

Start with a 45 degree right-angled triangle…

$$\sin 45° = \frac{1}{\sqrt{2}}$$

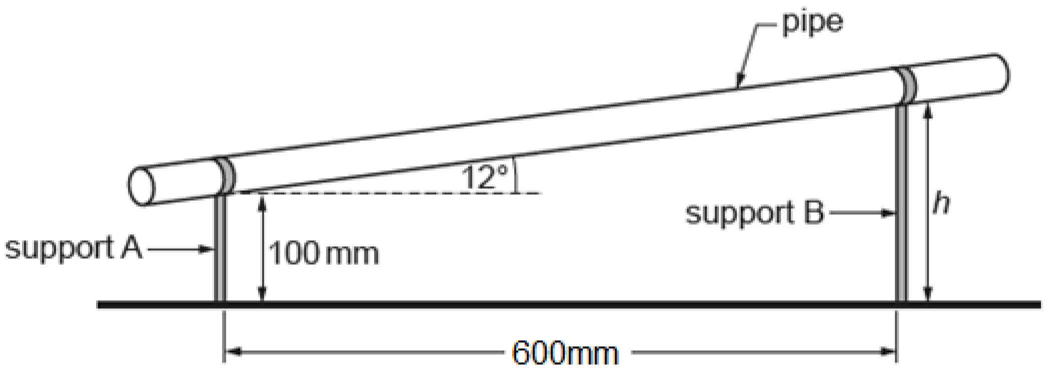
$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\tan 45° = 1$$



Practice 1

A design engineer wants to support a pipe at an angle of 12° to the horizontal to allow fluid to drain properly. The pipe is held between two vertical supports, 600 mm apart, as shown in the figure.

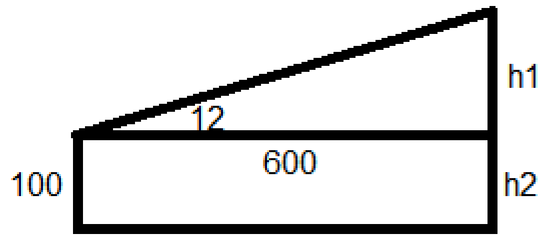


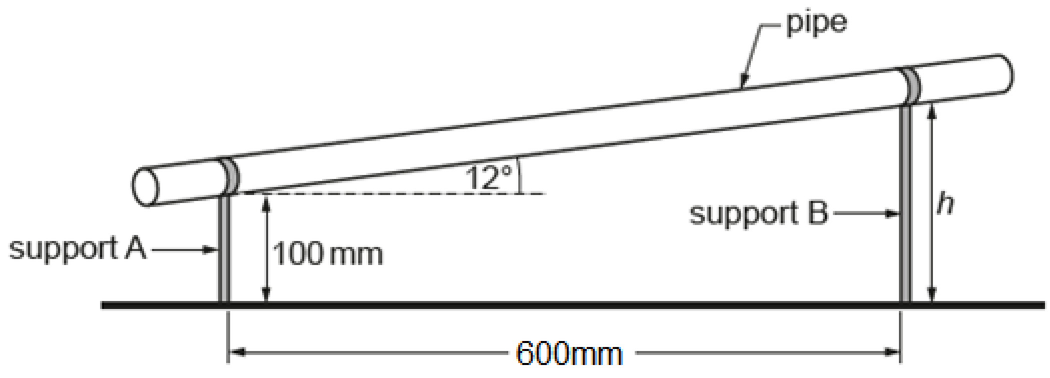
The height of support A is 100 mm.

Calculate the height (h) of support B. Show your working. [3]

Solution 1

This is a basic trigonometry problem.
Start by re-drawing the diagram more simply…





Tanθ = Opp / Adj
Tan12 = Opp / 600
600tan12 = Opp = **127.5**

We can see from looking at the diagram that H_2 is 100, so the overall height is 100 + 127.5 = **227.5mm**

Practice 2

A self-propelled vacuum cleaner calculates the time to clean a room based on the floor area.

The vacuum cleaner travels at a speed of $0.4ms^{-1}$.

In one particular rectangular room, the longest uninterrupted run is 12s. This run is assumed to be along the diagonal of the room from corner to corner. This room has a length to width ratio of 4:3.

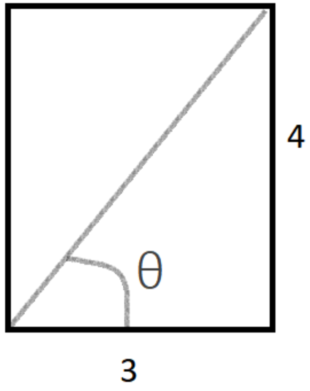
Calculate the floor area of this room. [5]

Source: OCR AS Specimen Paper, Q3d

Solution 2

3d. A self-propelled vacuum cleaner calculates the time to clean a room based on the floor area.
The vacuum cleaner travels at a speed of $0.4ms^{-1}$.
In one particular rectangular room, the longest uninterrupted run is 12s. This run is assumed to be along the diagonal of the room from corner to corner. This room has a length to width ratio of 4:3.
Calculate the floor area of this room. [5]
Source: AS Specimen Paper

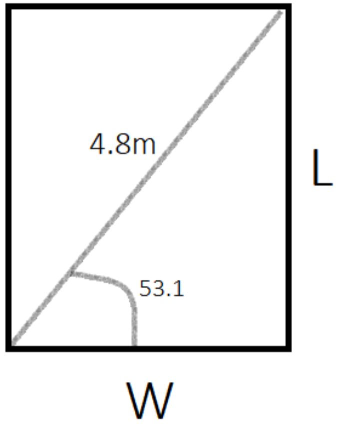
Step 1: Sketch the outline of the room.



As the ratio is known, we can calculate the angle θ…
tan θ = Opposite / Adjacent
tan θ = 4 / 3
Θ = tan-1(4/3) ≈ 53.1°

Step 2: Calculate the length of the longest run.
Speed = Distance / Time -> Distance = Speed x Time = 0.4 x 12 = **4.8m**

Step 3: Re-draw the sketch with what we now know,
and use trig to calculate the width and length of the room.



cos 53.1 = W / 4.8
4.8cos53.1 = W = **2.88m**

sin 53.1 = L / 4.8
4.8sin53.1 = L = **3.84m**

Step 4: Area = Length x Width
= 3.84 x 2.88 = **11.06m^2**

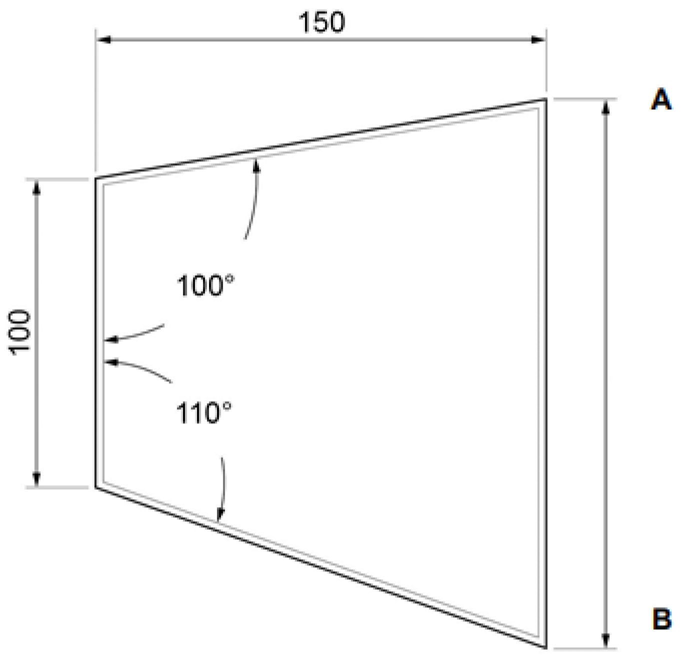
Practice 3

A lamp shade is formed to the profile shown in Figure 8.

Calculate the width of the shade along line AB.

[3 marks]

Source: AQA Product Design Specimen paper, Q8.3



All dimensions in millimetres

Solution 3

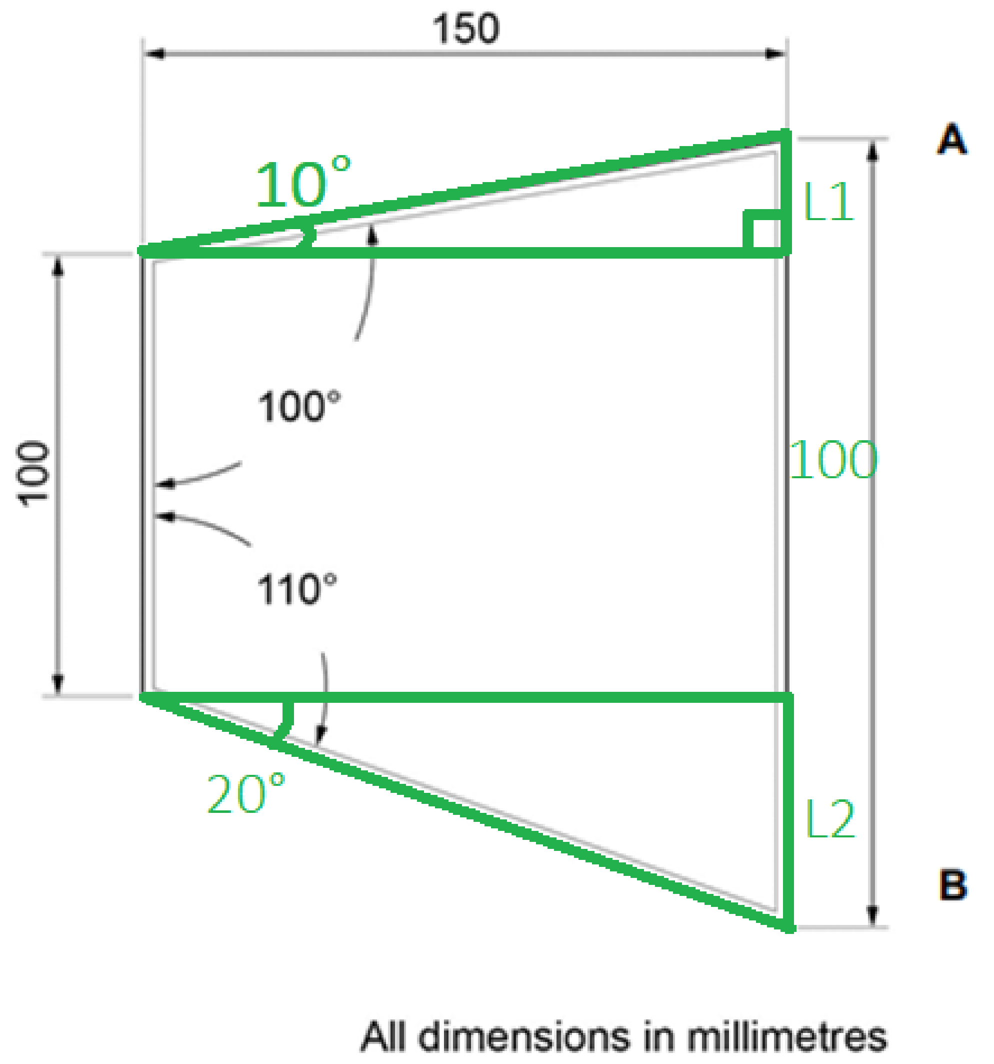
Answer = $L_1 + L_2 + 100$

$\tan\Theta = Opp / Adj$ $\quad$ $Opp = Adj \times \tan\Theta$

$L_1 = 150\tan 10 = 26.45mm$

$L_2 = 150\tan 20 = 54.60mm$

Answer = **181.05mm**



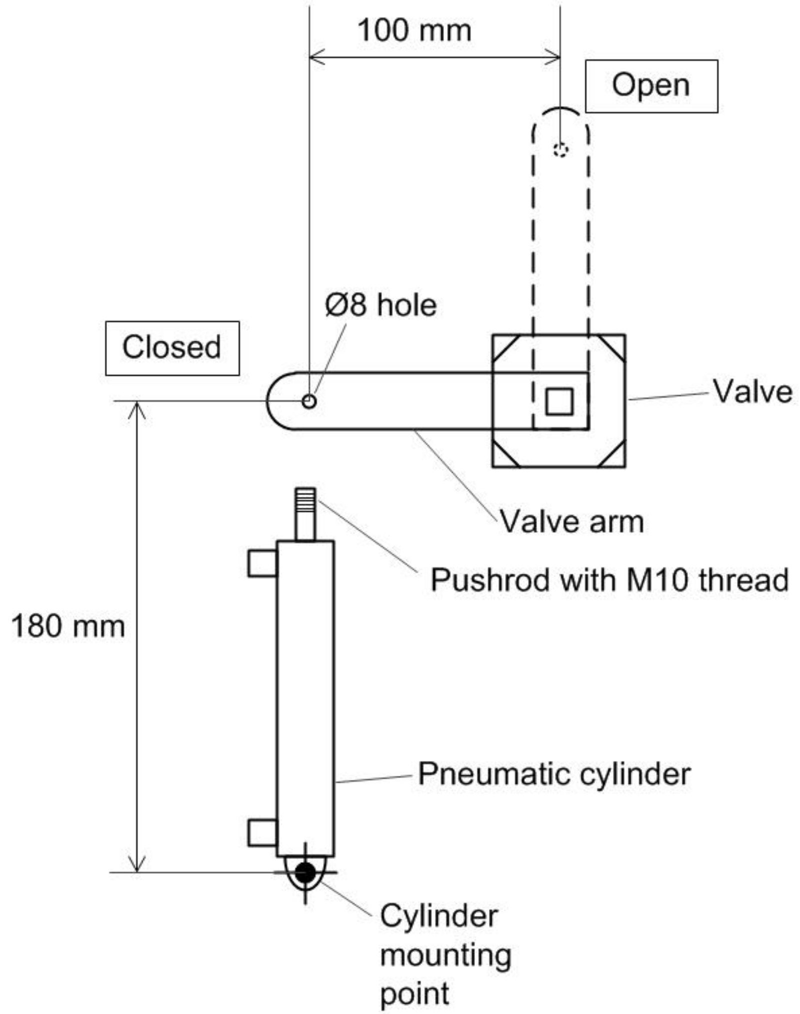
Practice 4

4. ii. A design engineer wishes to use a double-acting pneumatic cylinder as the actuator for a water valve.

The diagram shows the cylinder and valve in position. The valve is shown in the CLOSED position. When the cylinder out-strokes, it rotates the valve arm 90° to the OPEN position, shown by the dotted lines.

Analyse the data in the diagram to calculate, by a mathematical method, the pneumatic cylinder stroke required (stroke is the distance the pushrod extends when the cylinder is operated). You must show your working out. [5]

Source: AS Specimen Paper



Solution 4

Step 1: Draw a diagram to show and understand the problem.

The answer will be the difference between L_2 and L_1.

We already know L_1, which is handy.

Step 2: Use Pythagoras to calculate L_2.

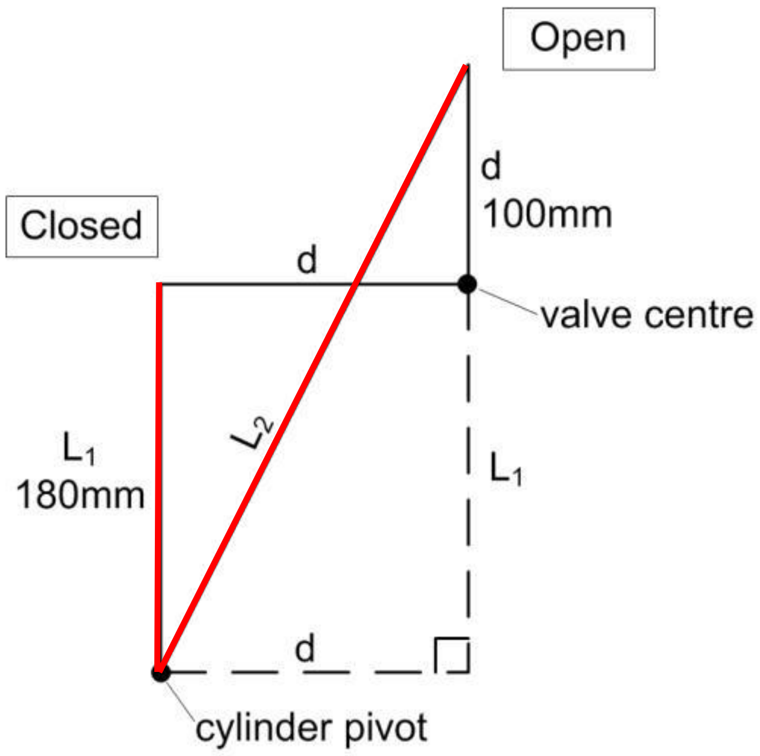
$a^2 + b^2 = c^2$

$100^2 + (180+100)^2 = c^2$

$88400 = c^2$ ...so $c = 297$mm

Step 3: Do $L_2 - L_1$ to get the answer.

297 – 180 = **117mm**



Practice 5

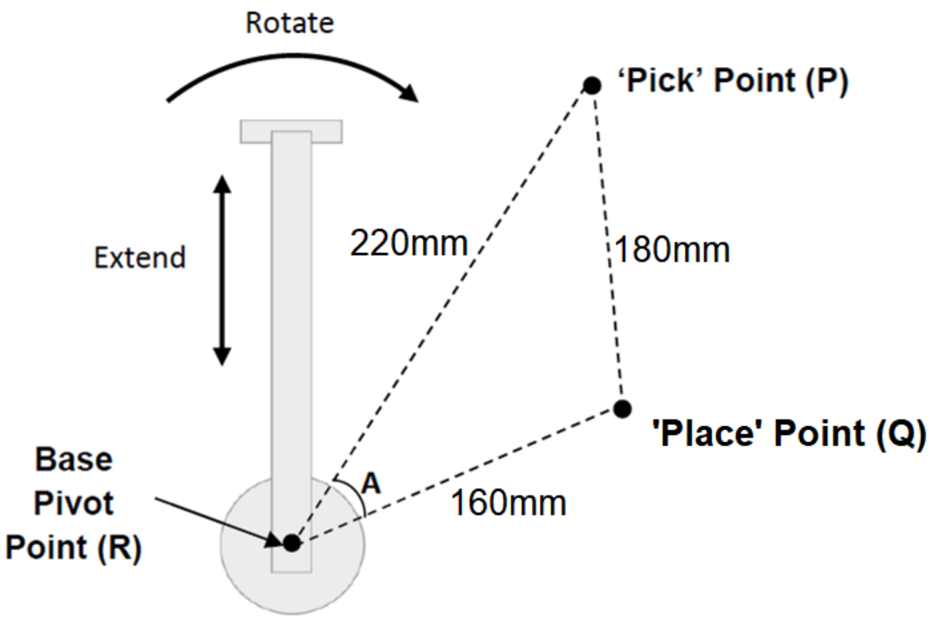
Robot arms are an example of automated technology.

The diagram shows the plan view of the arm of a pick and place robot. The arm can rotate and extend.

The robotic arm collects objects from Point P, before placing them at Point Q.

Calculate the angle A through which the robotic arm must rotate to move from Point P to Point Q. [5]

Source: Online example



Solution 5

No right-angles and no angles given.

Cosine rule to the rescue: $a^2 = b^2 + c^2 - 2bc.\cos A$

$180^2 = 160^2 + 220^2 - (2 \times 160 \times 220)\cos A$

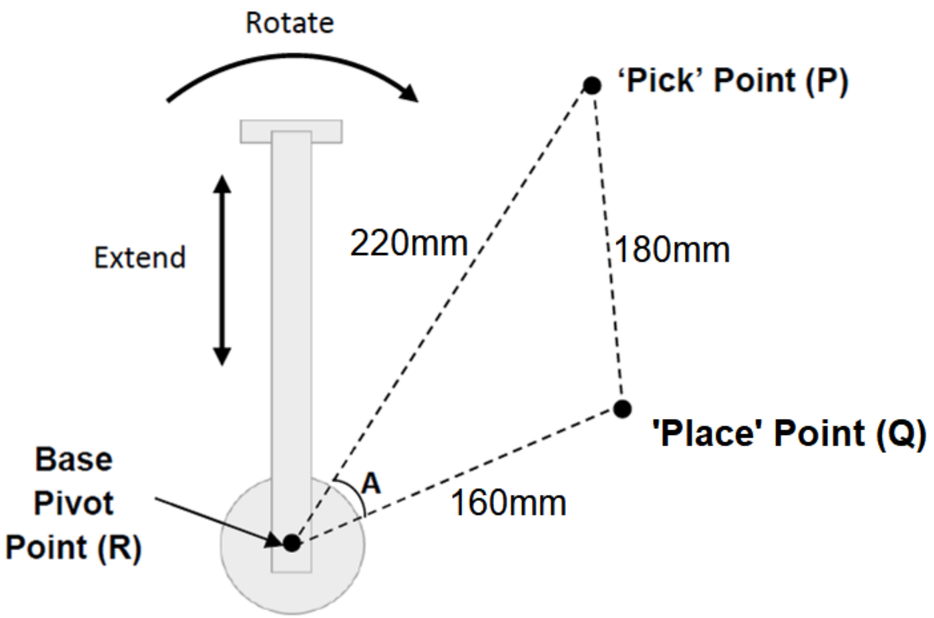
$32400 = 25600 + 48400 - 70400\cos A$

$32400 = 74000 - 70400\cos A$

$-41600 = -70400\cos A$

$-41600 / -70400 = \cos A$

$A = \cos^{-1} 0.591 = \mathbf{53.8°}$



Practice 6

2 (a) A light sensor is placed in a non-transparent tube so that it can only receive light from a specific light source.

Fig. 2.1 shows a two-dimensional side view of the tube which is 5 mm in diameter, placed perpendicular to the extended light source AB which is 100 mm wide.

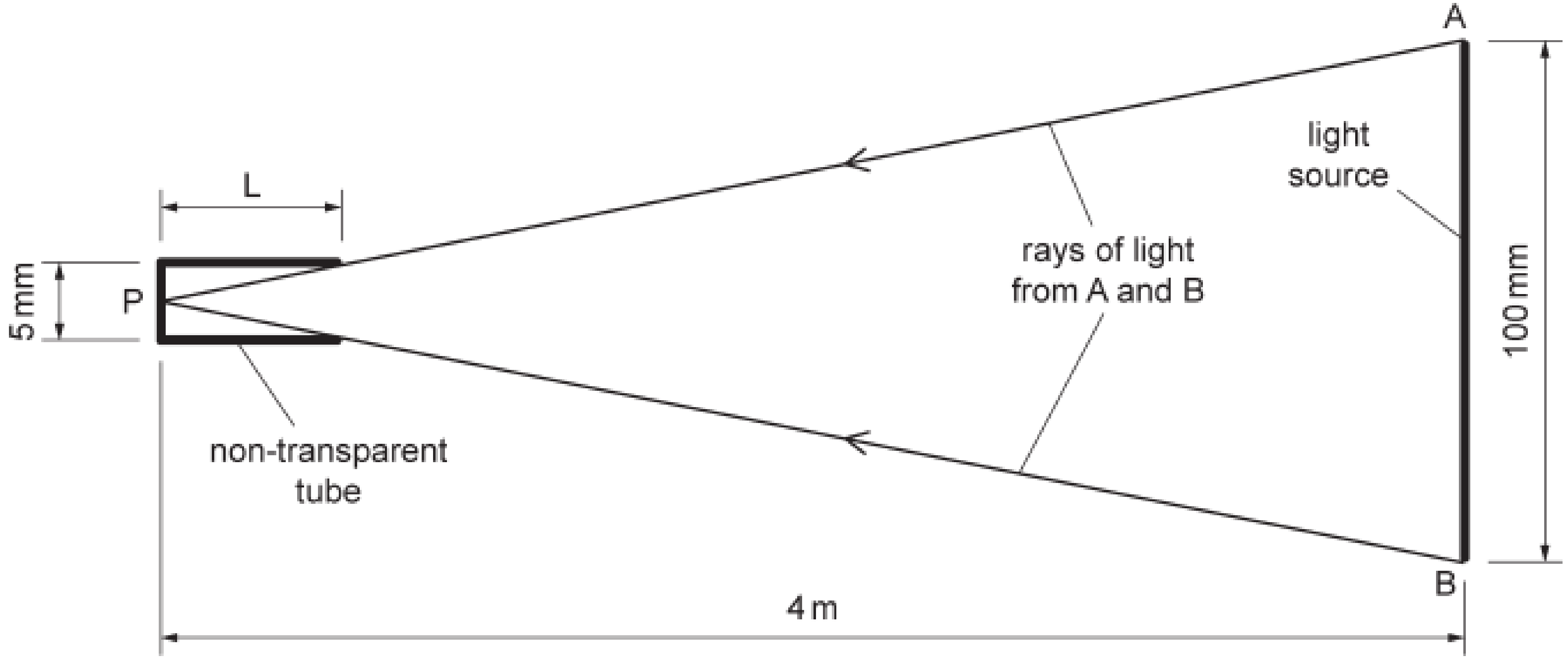


Fig. 2.1
(not to scale)

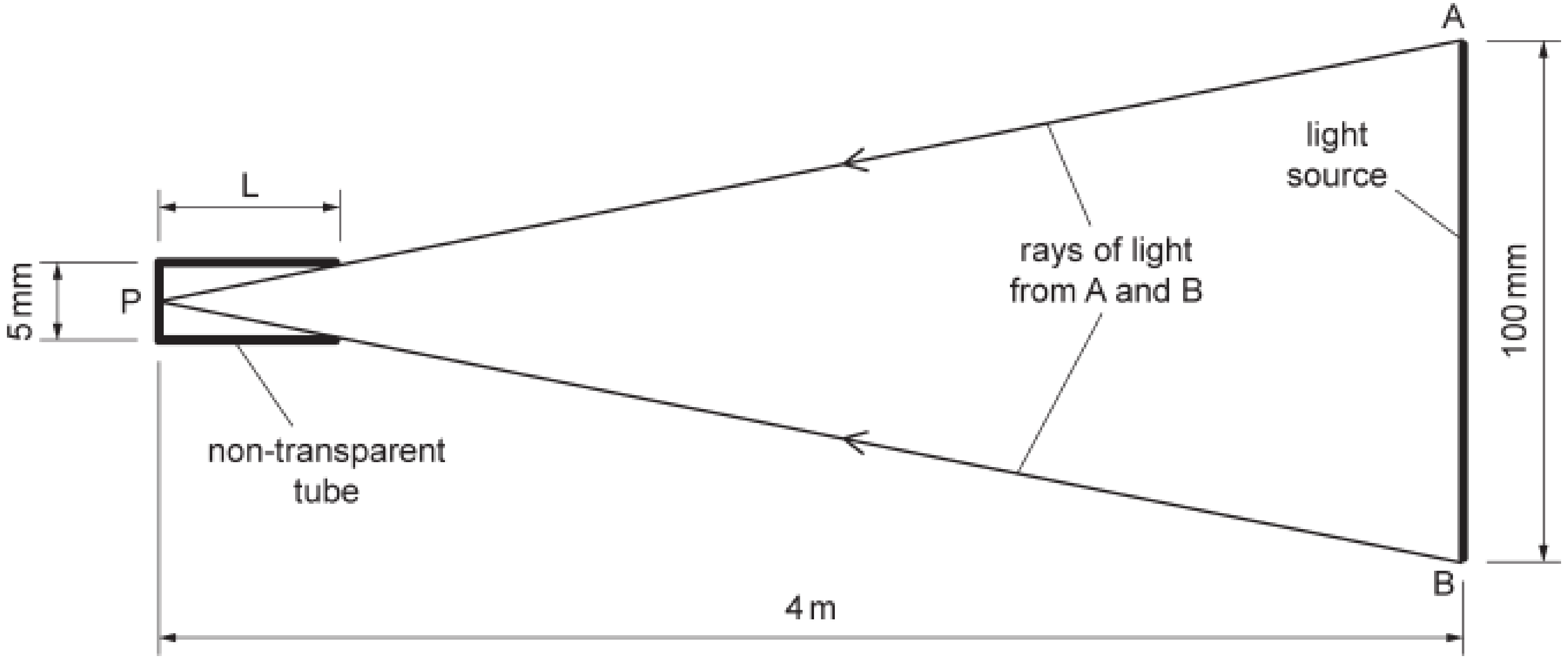
The light sensor will be placed at point P which is a perpendicular distance of 4 m from the light source AB.

Rays of light reaching point P from points A and B are drawn on **Fig. 2.1**.

Calculate the length L of tube required for this application. Give your answer in mm. Show your working.

Source: Paper 1 Q2a, 2020

Practice 6



4m = 4000 mm (1)

Similar triangles methodology: 100 / 4000 = 5 / L (1)

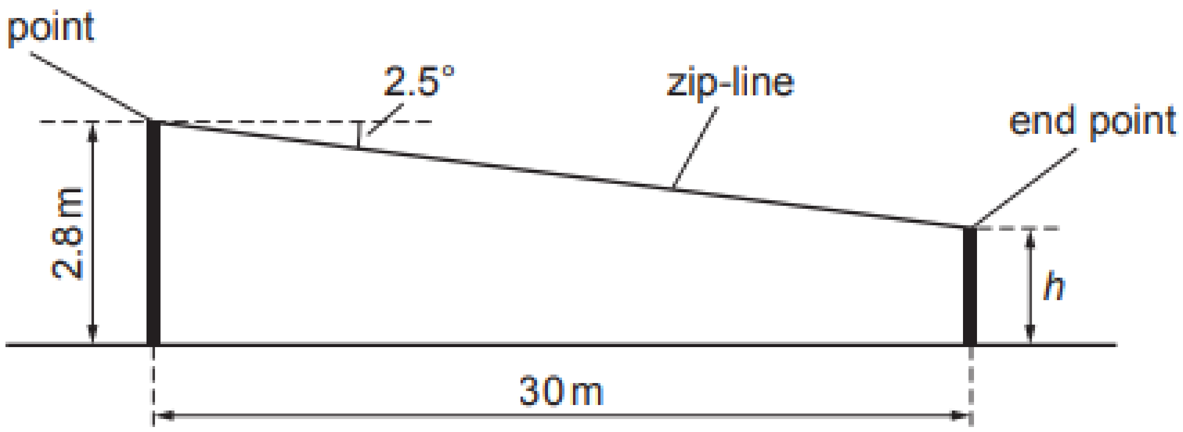
Finding L:

L = (4000 x 5) / 100

= 200 mm (1)

Source: Paper 1 Q2a, 2020

Practice 7

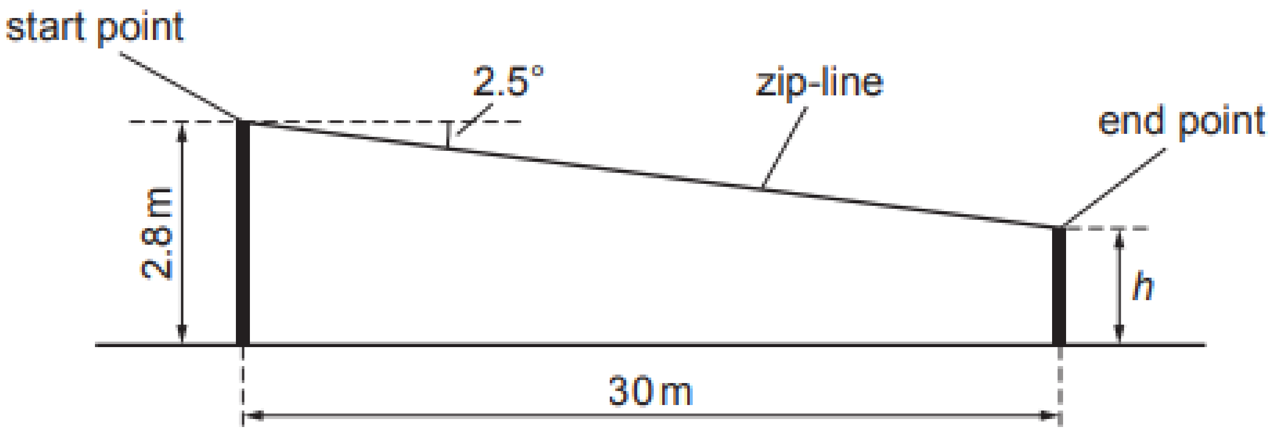


The diagram shows the basic design for a zip-line for a children's play park. Calculate the height *h* in m. Give your answer to 1 decimal place and show your working. Assume that the zip-line does not sag. [3]

Source: Paper 1 Q2ci, 2021

Solution 7

The diagram shows the basic design for a zip-line for a children's play park. Calculate the height *h* in m. Give your answer to 1 decimal place and show your working. Assume that the zip-line does not sag. [3]



tanΘ = Opp / Adj [1]

tan2.5 = Opp / 30

opp = 30 tan 2.5

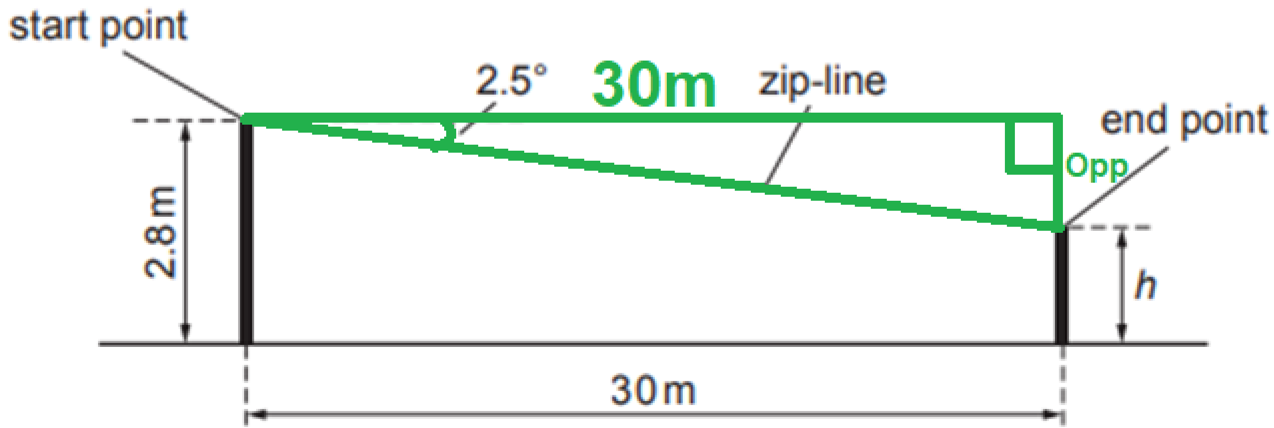
= 1.309828287m

= 1.3m [1]

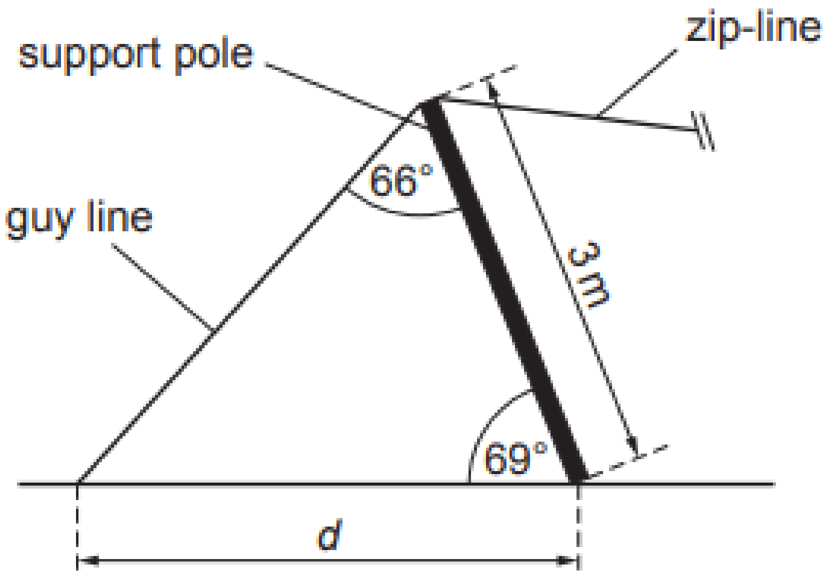
2.8 = opp + h

2.8 – 1.3 = h

h = 1.5m [1]



Practice 8



The diagram shows construction details of the start point for the zip-line.

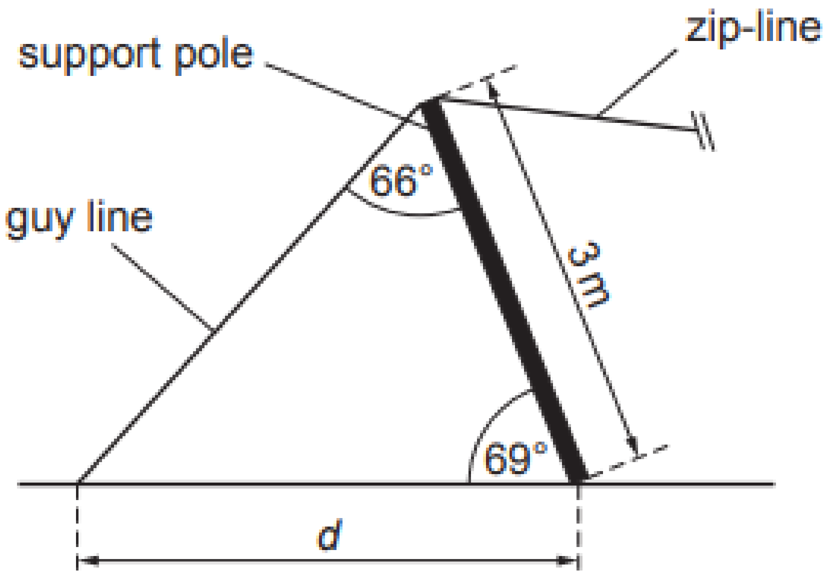
Calculate the distance *d* in m. Give your answer to 2 decimal places and show your working. [4]

Source: Paper 1 Q2cii, 2021

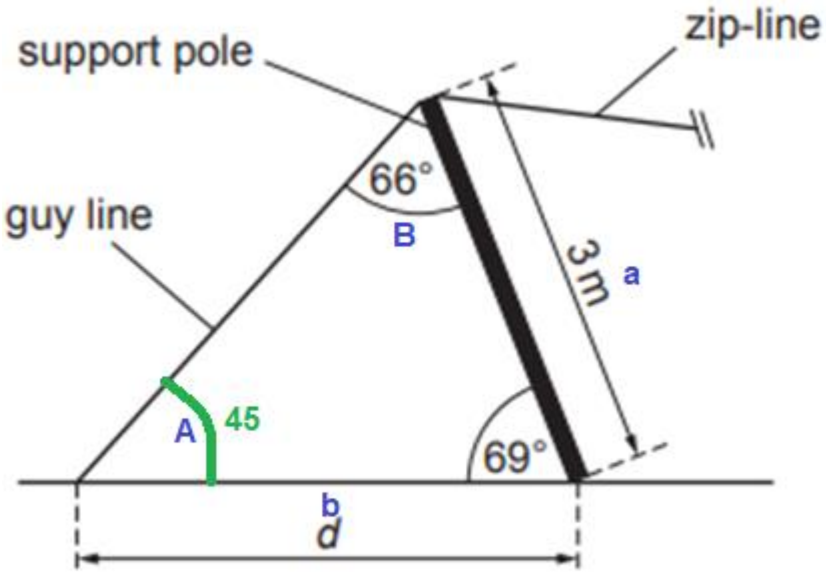
Solution 8

The diagram shows construction details of the start point for the zip-line.

Calculate the distance *d* in m. Give your answer to 2 decimal places and show your working. [4]



Calculate the remaining angle. 180 – 66 – 69 – 45 degrees. [1]



Sine rule:

a / sin A = b / sin B [1]

3 / sin45 = b / sin 66

b = 4.243 x 0.914 = 3.878102 [1]

b = 3.88m [1]

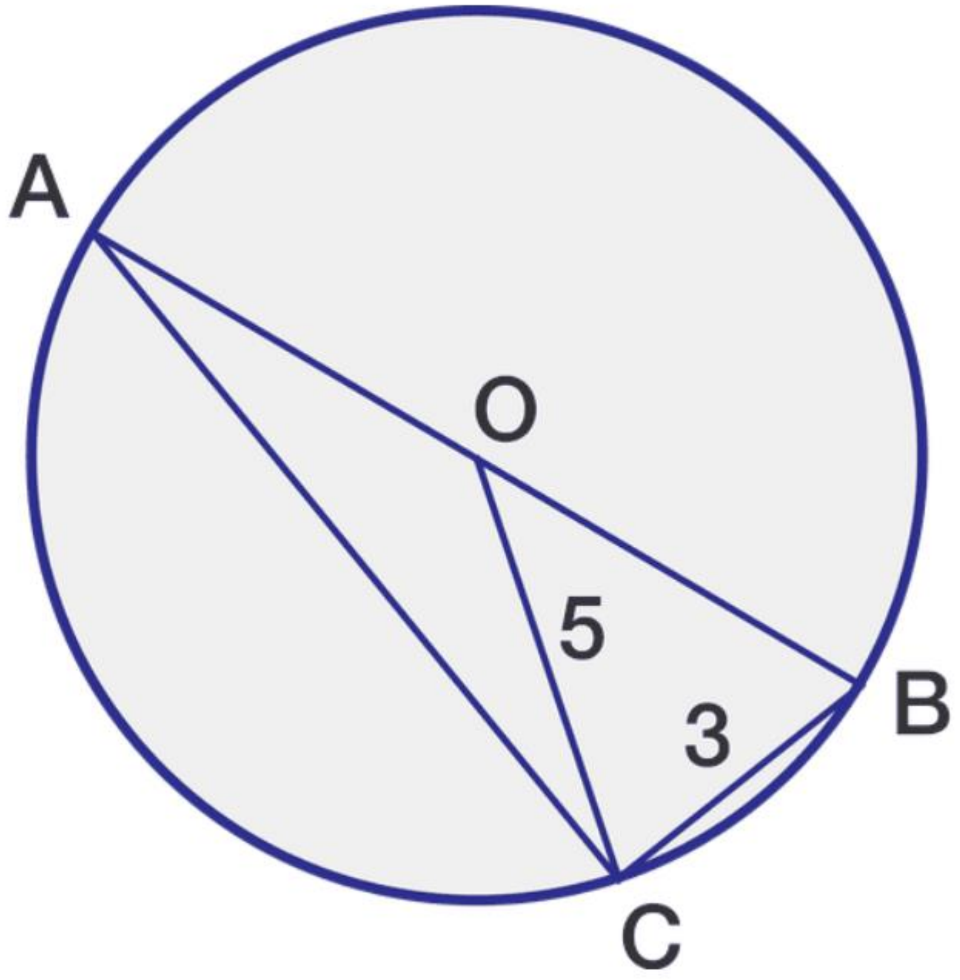
Practice 9

Robot arms can be used in 'pick and place' machines. Arm OB pivots about point O, has a fixed length of 5m and can sweep around a circular path. A simplified representation of this is shown in the diagram to the right.

An engineer wishes to know how far the arm travels point-to-point when it turns from position OC to OA.

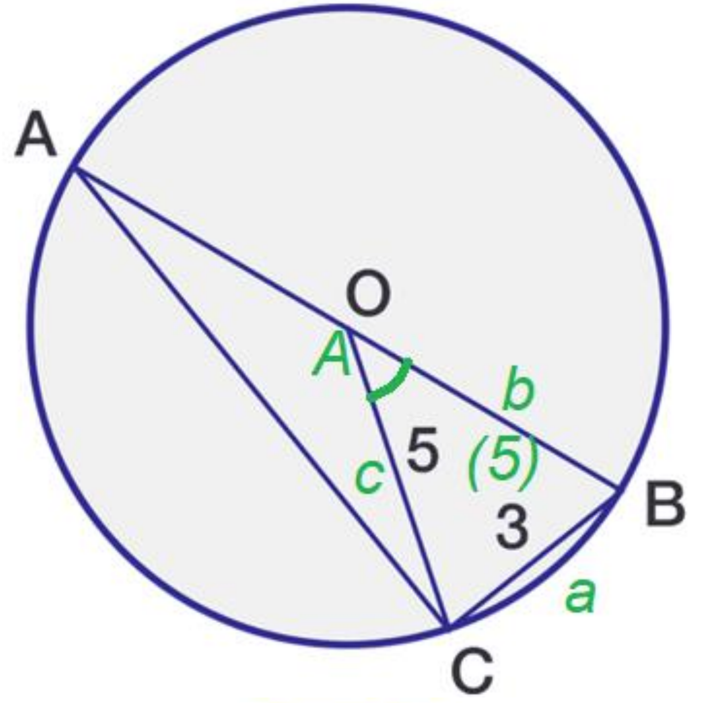
Given that AB is the diameter of the circle shown, what is the value of AC?
[4]

Source: Online example, adapted.



Solution 9a

Strategy: If we can find angle BOC, we can subtract it from 180 to give angle AOC. That can then be used with some trigonometry to find line AC.

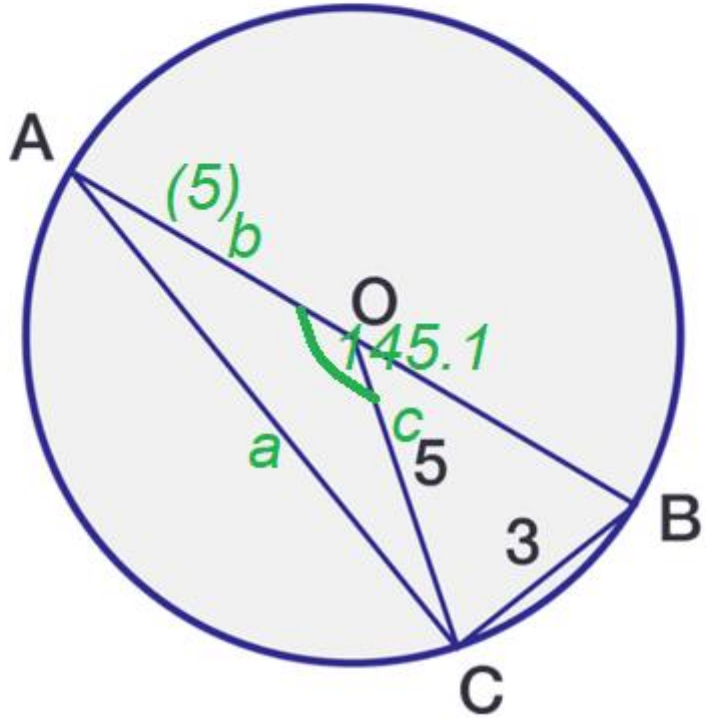


1. Cosine rule: $a^2 = b^2 + c^2 – 2bc.\cos A$

2. Re-arrange: $b^2 + c^2 - a^2 - 2bc.\cos A = 0$

$b^2 + c^2 - a^2 = 2bc.\cos A$

$(b^2 + c^2 - a^2) / 2bc = \cos A$



3. Plug in values: $(25+25-9) / (2x5x5) = 41/50 = 0.82.$ $\rightarrow A = 34.9°$

$AOC = 180 – 34.9 = 145.1°$

4. Cosine rule (again): $a^2 = b^2 + c^2 – 2bc.\cos A$

$a^2 = 25+25 – (2x5x5)\cos 145.1$

$a^2 = 50 – 50\cos 145.1 = 50 – –41.0 = 91$

a ≈ 9.54m

Trigonometry: Solution 9b *(alternative)*

Strategy: Circle theorem tells us that the angle subtended by an arc at the centre is twice the angle subtended at the circumference.

AOB = 180 degrees

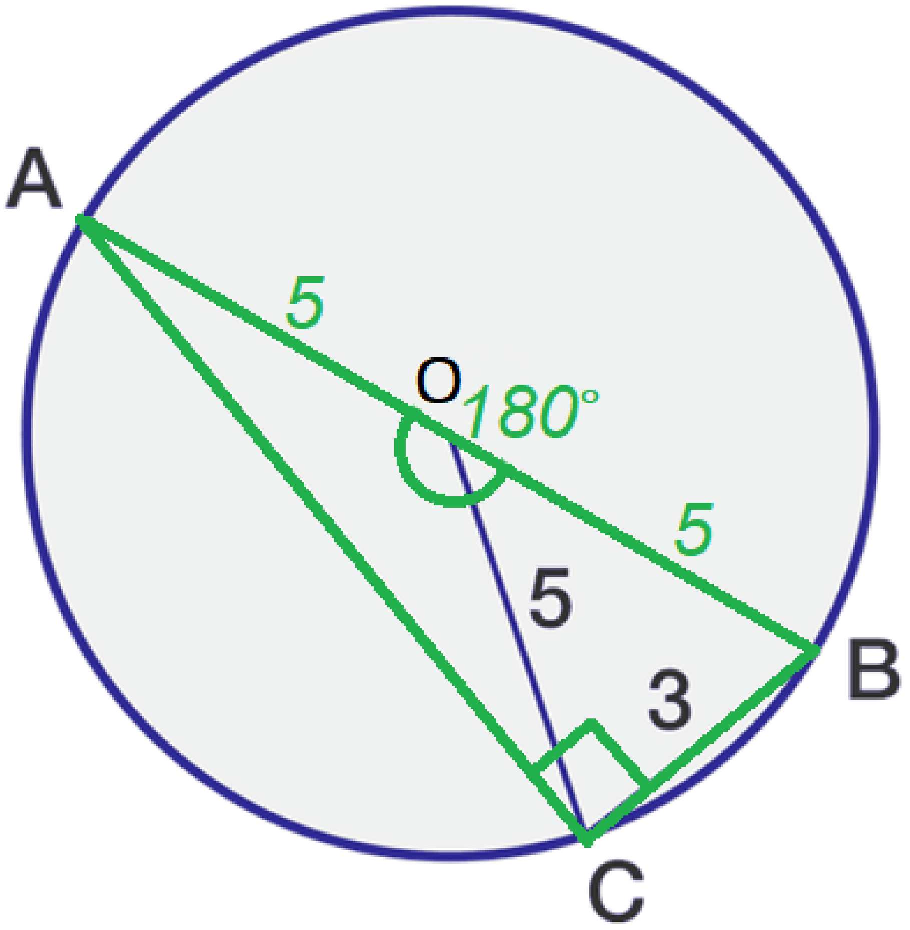
ACB = 180 / 2 = 90 degrees. (*i.e. no matter where C sits, it will always be 90*)

We can treat the shape as a right-angled triangle and solve with Pythagoras.

$3^2 + b^2 = 10^2$

$b^2 = 100 - 9 = 91$

b = sqrt(91) ≈ **9.54m**



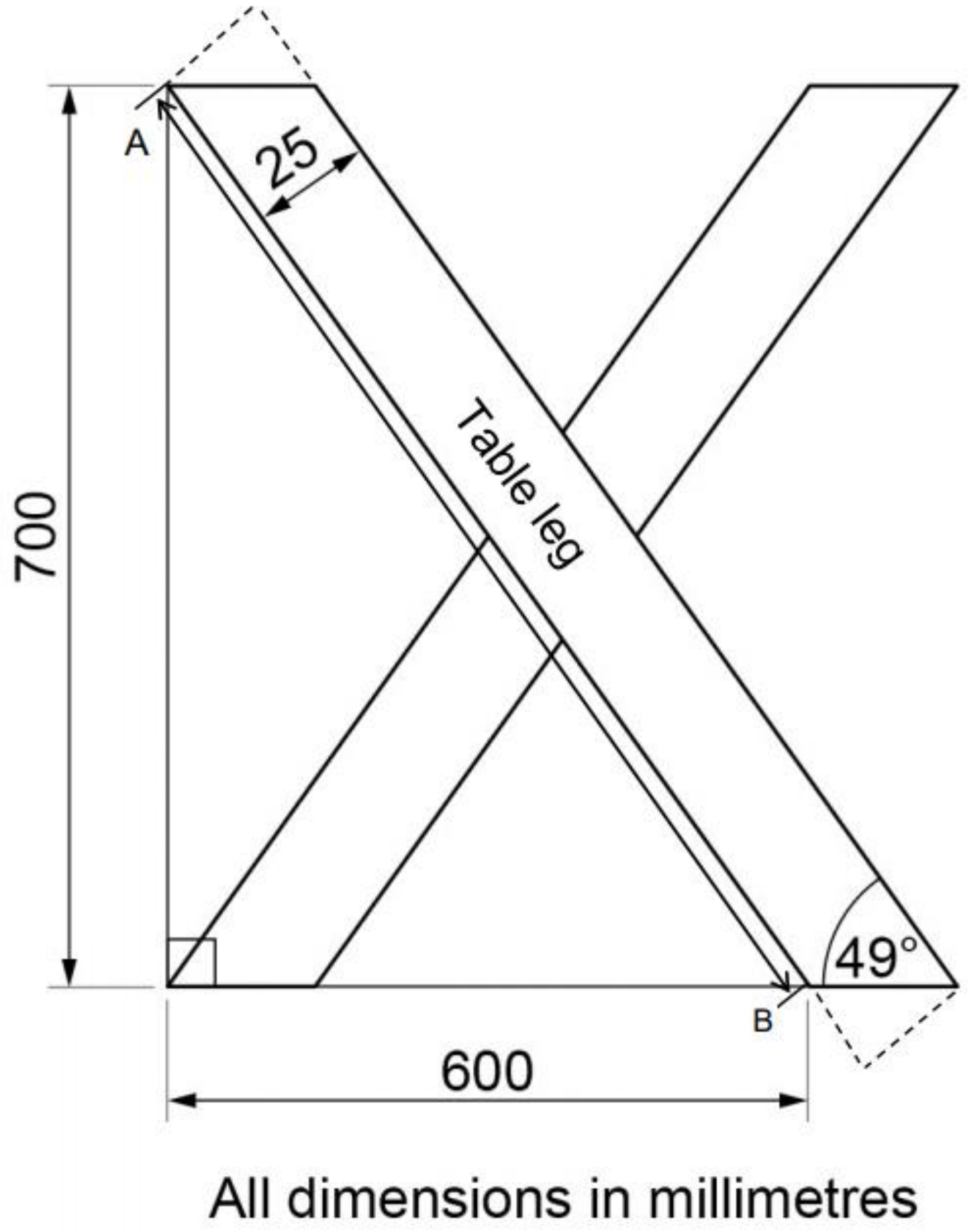
Practice 10

A dining table has been manufactured from lengths of wood. The figure shows a cross sectional view of **two** of the table legs.

i. Determine the length AB shown in Figure 4

ii. Calculate the minimum total length of a single piece of wood required to manufacture the four legs of the table, assuming no wastage from the cutting processes. Ignore blade thickness for the purpose of the calculation

Source: AQA Product Design Specimen paper, Q5



Solution 10

Main piece of wood is a rectangle.

$a^2+b^2 = c^2$ $600^2 + 700^2 = c^2$ c = sqrt(360000 + 490000) = **921.95mm**

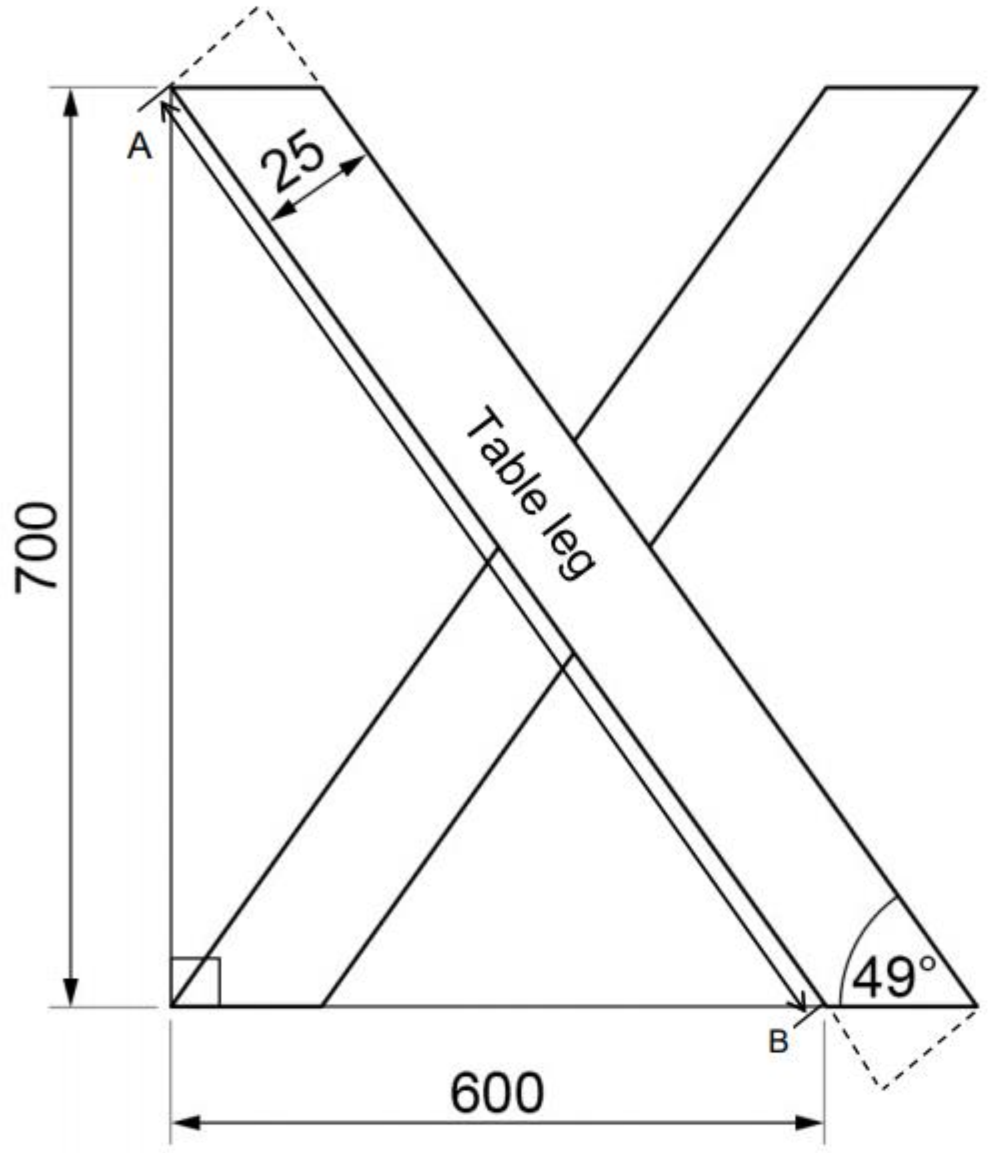
There's an extra triangular piece on the end, where the 49° angle is cut.

tanΘ = Opp / Adj So 25 / tan49 = Adj = **21.73mm**

i. Length of leg = 921.95 + 21.73 = 943.69mm

ii. Overall length is (921.95 x 4) + 21.73 = **3709.53mm** (3.71m)

Source: AQA Product Design Specimen paper, Q5



All dimensions in millimetres

Practice 11

The stem of a lamp is bent from a single tube of mild steel with the dimensions shown in Figure 8.

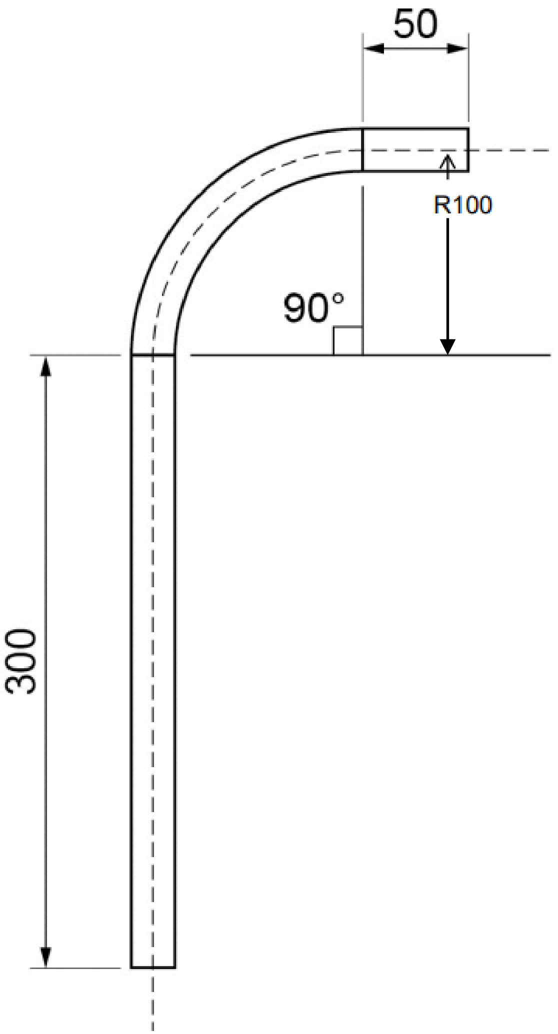
Calculate the length of tube required to create the stem. Give your answer to the nearest mm.

BONUS: Describe how the tube could be formed. In your answer you should: explain the forming process and refer to the tool(s) used.

[6 marks]

Source: AQA Product Design Specimen paper, Q8.2

Figure 8



Solution 11

i. Total length is 300mm + 50mm + a quarter of a circle formed with a 100mm radius.

$C = 2\pi r$ $= 2 \times \pi \times 100 = 628.32$mm $628.32/4 = 157.08$mm

300+50+157.08 = **507.08mm (507mm to the nearest mm)**

BONUS. A pipe bender is used: The tube is bent round a former with a lever and force is applied to create the bend - mechanical advantage afforded by the lever. This would only work if the tube is reasonably small (E.g. 1mm thick), and the pipe would need inspecting to ensure it doesn't get crushed. When the angle is being set, an additional few degrees would need to be added to compensate for the 'springback' that occurs when removing the bent pipe. (*Included for info - not part of the specification*)

Trigonometry: Further Reading

http://www.mathcentre.ac.uk/resources/uploaded/mc-web-mech1-5-2009.pdf

- Force as a vector; worked examples and practice questions

https://www.cimt.org.uk/projects/mepres/step-up/sect4/index.htm

- Sine and cosine rule. Exercises at bottom of the page.